Introduction

Failure prediction and description of medium voltage cables reliable life is very important part of the distribution network operation plans. Weibull probability distribution function is very often used to describe reliability of myriad various products all over the world. The calculation method of Weibull reliability parameters of MV cables comes from knowledge of failure rates for each period of cables lifetime. Availability of both failure database differed by cable’s production year and actual distribution of cable’s age is essential for this analysis.

Weibull probability distribution

The Weibull distribution is one of the most widely used lifetime distributions in reliability engineering. It is a versatile distribution that can take on the characteristics of other lifetime distributions in reliability engineering. It is a versatile

\[ F(t) = 1 - e^{-\left(\frac{t}{\eta}\right)^\beta} \]

where \( \beta \) is shape parameter of Weibull distribution, \( \eta \) is scale parameter (so called characteristic life) and \( \gamma \) is location parameter.

Fig.1. Cable age histogram

Age distribution of MV cables

Age distribution of MV cables of investigated part of electrical distribution network consists of cable lengths of specific production year. Example of age distribution (also called age histogram of cables) is shown in Figure 1.

Distribution period has not significant influence on calculation. We have done at VŠB-TU of Ostrava calculations using both one and five year period study and the results were similar.

Failure rate function \( \lambda(t) \)

Failure rate is defined as ratio of number of failures in studied file of components to file size. In our case the file of components are all MV cables in studied network of certain age (or also of certain type) and the failure number we can obtain out of failure database. Failure rate function \( \lambda(t) \) is then set of point values of particular failure rates calculated according to equation [1]:

\[ \lambda_i = \frac{n_i \cdot 100}{l_i \cdot p} \] (1/year/100 km)

where: \( n_i \) is number of failures of \( i \)-year old cables, \( l_i \) is total length of \( i \)-year old cables in network (km), \( p \) is studied period (year).

Failure rate value is in failures per annum per 100 km of cable length because it is usual unit for power lines in the Czech Republic. Follows example of calculated failure rate function (Figure 2):
The line in the Figure 2 represents linear regression calculated failure rate values. Due to more accurate results we have used weighted regression based on length of cable of specific age – the longer the cables of specific age are the greater influence on regression curve they have. So the weights for weighted regression were values from cable age histogram (Figure 1).

Also the length of error abscissas in Figure 2 is derived from cable length as reciprocal value. The longer abscissa is the less trustworthy failure rate value is calculated.

Regression models

When we need to describe cables reliability using Weibull parameters we have to interleave failure rate values with a weighted regression curve which corresponds to failure rate function of Weibull distribution [1].

Two regression models were employed – Linear and Power function. Weibull reliability parameters can be easy calculated out of parameters of both these regression models.

The easiest model as regards backward transformation of regression to Weibull parameters is the power function

\[ \lambda(t) = a \cdot t^b \]

Let us compare this equation with failure rate of two-parameter Weibull distribution:

\[ \lambda(t) = a \cdot t^b \]

It is obvious that if this equation should stand the powers on both sides must be equal:

\[ b = \beta - 1, \text{ or } b = \beta + 1 \]

Similarly we can deduce the shape parameter:

\[ \beta \]

The linear regression model consists of two portions of failure rate functions: constant and pure linear. The regression function is:

\[ \lambda(t) = a + b \cdot t \]

The constant portion \( a \) represents random failures of cables (consistent with exponential distribution \( a = \lambda \), or Weibull \( a = 1/\eta \) with shape parameter \( \beta = 1 \)). The linear portion is caused by cables wear-out, the failure rate grows linear with time. That is characteristic for Rayleigh probability distribution, or also Weibull distribution with the shape parameter \( \beta = 2 \). Also the failure rate is:

\[ \lambda(t) = a + b \cdot t \]

The exponential parameter \( \lambda \) and Weibull shape parameter \( \beta \) were already discussed, only the Weibull scale parameter \( \eta \) is left to deduce:

\[ b = \frac{2}{\eta^2} \Rightarrow \eta = \sqrt{\frac{2}{b}} \]

The linear regression model with sum of two portions of failure rate functions forms series connected reliability scheme with two components described by two different reliability functions. The reliability scheme is:

\[ \begin{array}{cc}
1 & 2 \\
\lambda &= \text{const} \\
\lambda(t) &= \frac{2}{N} \cdot t
\end{array} \]

Fig. 2. Reliability model of cable line characterized by linear failure rate function

Length dependent reliability

The reliability of energy lines (both cables and overhead) contrary to single components depends on line length. The longer line is the more failures will occur. Considering Weibull probability distribution the recomputation of reliability parameters for specific length of line is not as simple as in case of exponential distribution.

Further we will deal only with MV cables (recomputation for overhead lines is the same). We will depict deduction of length dependence on an example. Each cable we can imagine as \( N \) sections of particular length \( L_{ref} \) connected in series. Whole cable length is then [2]:

\[ L = L_{ref} \cdot N \]

When the particular lengths are equal then the total cable reliability is:

\[ R_{\text{CABLE}} = R_1 \cdot R_2 \cdot \ldots \cdot R_N = R^N \]

When we assume Weibull distribution function for each particular section:

\[ R_{\text{CABLE}}(t) = \left( e^{-\left(\frac{t}{\eta}\right)^\beta} \right)^N = e^{-\left(\frac{t}{\eta}\right)^\beta} \]

Consequent to above described cable reliability we need one more parameter to generalize Weibull failure model for arbitrary cable length, the reference length \( L_{ref} \). In accordance with electric power reliability in the Czech Republic the reference length is chosen \( L_{ref} = 100 \text{ km} \). Unlike at exponential distribution the Weibull shape parameter reciprocal \( 1/\eta \) is not directly proportional to cable length \( L \). The deduction of shape parameter recalculation for arbitrary length follows:

\[ e^{-\left(\frac{t}{\eta_{\text{ref}}}\right)^\beta} = e^{-\left(\frac{t}{\eta}\right)^\beta} \]

\[ \eta = \frac{\eta_{\text{ref}}}{\sqrt{N}} \]

Where: \( \eta_{\text{ref}} \) is desired Weibull shape parameter for length \( L \), \( \eta_{\text{ref}} \) is desired Weibull shape parameter of \( L_{ref} = 100 \text{ km} \) long power line and \( N = L/L_{ref} \) is desired length to reference length ratio.

Results

Cable reliability study using methodology depicted in this paper has been done for two electricity distribution utilities. The main problem of calculation was the quality of source failure database. Not all of failure records have had
production/installation year of damaged cable included. Production dates were recorded by 93 % for utility with code name REASM and by 36 % for utility REASP.

At REASM the non-weighted regression was performed with no constant portion, because calculation led to negative value of $a$.

Table 1. Results of REASM study

<table>
<thead>
<tr>
<th>Weighted Power Regression</th>
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</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>Beta</td>
</tr>
<tr>
<td>Eta ($L_{ref}=100km$)</td>
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Table 2. Results of REASP study

<table>
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Linear Regression for $a = 0$ - Nonweighted

<table>
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<tr>
<th>Constant portion:</th>
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<tbody>
<tr>
<td>$\lambda$ = 0</td>
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Weibull/Rayleigh portion:

| Beta = Const. | 2 | - |
| Eta ($L_{ref}=100km$) | 3.291 | Year |

Calculation verification

Next step for all studies should be verification of calculated values. For comparison of results and thus evaluation the calculated failure rate was confronted with source database. For each period (1 or 5 years) the number of failures was calculated for each cable age group and the results were confronted with actually recorded number of failures. Moreover, the life interval of cables (approx. 46 years) was split into halves. The first represents mostly useful life of cable life, the second is the end of useful life. Regression models (weighted power and linear) were further compared with model with solely exponential distribution, those parameter was calculated out of failure databases of several Czech and Slovak utilities ($\lambda=4.85 1/$Year/100 km).

Table 3. Calculation verification for REASM - example

| Comparison of calculation models with failure number from database – Sum of failures |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|
|                                | Database | Power Weight. | Linear. Weight. | Exp. $\lambda=4.85$ |
| Sum                            | 524      | 507.9          | 538.7           | 565.3           |
| 1. half interval               | 322      | 315.3          | 324.7           | 413.7           |
| 2. half interval               | 202      | 192.6          | 212.0           | 151.6           |

Comparison of calculation models with failure number from database – Relative values (%)

|                                | Database | Power Weight. | Linear. Weight. | Exp. $\lambda=4.85$ |
| Sum                            | -        | -3.07         | 2.42            | 7.88             |
| 1. half interval               | -        | -2.08         | 0.83            | 28.49           |
| 2. half interval               | -        | -4.66         | 4.95            | -24.97          |

Conclusions

Most of the work presented in this paper was done in terms of Ph.D. thesis of one of the authors [1]. However, the response from concerning utilities is mostly positive, further implementation of this method is view of end of 2007. The biggest problem is connected with quality of source database – the ratio of production year records. Hopefully the results of this project will convince also other utilities to take more care about filling out production date records and to provide information about age distribution of cables.

This method is also useful for other electrical distribution network components but the influence of maintenance must be taken into account.

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REFERENCES


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